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k 组合数通连 α、β 变比热解

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文献 [1] 导出了以比热比 k 表示的变比热气动函数式，并根据焓值拟合式给出了解析解。文献 [2] 用以求解内外函混合。文献 [3] 给出以三个 α 系数图解曲线为基础的仍是以速度系数 λ 为自变量的变比热气动函数式。文献 [4] 开展了以两个 β 系数为基础的以马赫数 Ma 为自变量的变比热气动函数式，并都给了算例。本文试图以几个 k 组合数 ε 来通连 α、β 系数于变比热气动函数式及其解析解，使实际应用更为方便。

1. 以 k 组合数 ε 简化变化比热气动函数 解析解用到的 k 分别是[1]：

$$k_T/(k_T-1)\cdot R = c_p = i_{T+.5}-i_{T-.5}\quad,\quad k_k/(k_k-1)\cdot R = c_{pk} = i_{T_k+.5}-i_{T_k-.5}$$

$$k/(k-1)\cdot R = \overline{c_p} = (i^*-i)/(T^*-T)\quad,\quad k_v/(k_v-1)\cdot R = \overline{c_{pv}} = (i^*-i_k)/(T^*-T_k)$$

$$\overline{k}/(\overline{k}-1) = \lg(\pi_T^o/\pi_{T^*}^o)/\lg(T^*/T)\quad,\quad \overline{k}_v/(\overline{k}_v-1)\cdot R = \lg(\pi_{T^*}^o/\pi_{T_k}^o)/\lg(T^*/T_k)$$

只要引用 k 组合数

$$\varepsilon_T = (k-1)/2\cdot k_T/k\quad,\quad \varepsilon = (k-1)/2\cdot k_k/k_v\quad,\quad \varepsilon_v = 1+(k_v-1)/2\cdot k_k/k_v \tag{1}$$

就可把原来以各个 k 表示的气动函数简化如下：

临界温比 $T_k/T^* = 1/\varepsilon_v$，速度系数 $\lambda = c/\sqrt{k_k/(RT^*\varepsilon_v)}$，静总温比 $\tau = 1-(\varepsilon/\varepsilon_v)\lambda^2$，静总压比 $\pi = \tau^{\overline{k}/(\overline{k}-1)}$

在流量公式 $\dot{m} = F(p^*/\sqrt{T^*})mq$ 中的系数 m 简化为：$m = \sqrt{k_k/R}(1/\varepsilon_v)^{(\underline{k}_v+1)/2(\underline{k}_v-1)}$，而密度流比是：$q = \rho c/\rho_k a_k = \lambda\tau^{1/(\underline{k}-1)}\varepsilon_v^{1/(\underline{k}_v-1)}$

如按文献 [3] 所用流量公式，并为区别清楚，以 q 表示其函数 $\dot{m} = F(p^*/\sqrt{T^*R})\underline{q}$

则可得

$$\underline{q} = \sqrt{R}mq = \lambda\tau^{1/(\underline{k}-1)}\sqrt{k_k/\varepsilon_v} \tag{2}$$

有趣的是这种形式的 $\underline{q}$ 函数竟将 $\overline{k}_v$ 消去。然它已不是单纯的密流比。

在冲量公式 $S = \dot{m}[(k+1)/(2k)]Q_k z = F^*{}_p^* f$ 中的 z、f 函数简化成

$$z = \lambda + 2k\varepsilon_v/(k+1)k_k\cdot 1/\lambda\quad,\quad f = (k+1)/2k\cdot k_k(1/\varepsilon_v)^{\overline{k}_v/(\overline{k}_v-1)}qz \tag{3}$$

如按文献 [3] 所用冲量公式，并为区别清楚，以 $\underline{z}$ 表示其函数：$S = \dot{m}\sqrt{RT^*}\underline{z} = F_p^* f$，则可得

$$\underline{z} = (k+1)/2k\cdot\sqrt{k_k/\varepsilon_v}\,z\quad,\quad f = \dot{m}(\sqrt{RT^*}/F_p^*)\underline{z} = \sqrt{R}mq\underline{z} = \underline{q}\,\underline{z} \tag{4}$$

这里看出 $\underline{q}$ 与 $\underline{z}$ 乘积即是 f 函数的简明关系。这在文献 [3、4] 中均已引证。

与马赫数有关公式是：

$$\tau = 1/(1+\varepsilon_T Ma^2)\quad,\quad Ma^2 = k_k/k_T\varepsilon_v\cdot\lambda^2/\tau \tag{5}$$

至此主要变比热气动函数简化得已够清新的了。如以定比热时 $\varepsilon_T = \varepsilon = (k-1)/2$、

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$\varepsilon_v = (k+1)/2$ 代入，各式均还原为定比热气动函数。

2. 以 k 组合数 ε 通连 α 系数气动函数式　文献 [3] 所用 α 系数，都可分析如下：

$$\alpha_a = \sqrt{k_k R T_k / T^*} = \sqrt{k_k R / \varepsilon_v}$$

$$\alpha_h = R k_k / 2 \cdot T_k / T^* \cdot (T^* - T)/(i^* - i) = (k-1)/2 \cdot k_k / k \cdot 1/\varepsilon_v = \varepsilon / \varepsilon_v$$

$$\alpha_s = R \ln(T^* / T) / R \ln(\pi_T^{o*} / \pi_T^o) = (\bar{k} - 1)/\bar{k} \tag{6}$$

导出 α 系数则是：

$$\alpha_* = \alpha_a^2 / \sqrt{R} = \sqrt{k_k / \varepsilon_v} \tag{7}$$

$$\alpha_l = \alpha_*^2 - \alpha_h = (k_k - \varepsilon)/\varepsilon_v = k_k \left[1 - (k-1)/2k\right]/\varepsilon_v = \left[(k+1)/2\right](k_k / k)/\varepsilon_v$$

于是各气动函数是：$\tau = 1 - \alpha_h \lambda^2 = 1 - (\varepsilon / \varepsilon_v)\lambda^2$ ，　$\pi = \tau^{1/\alpha_s} = \tau^{\bar{k}/(\bar{k}-1)}$

$$\underline{q} = \alpha_* \lambda \tau^{(1/\alpha_s) - 1} = \sqrt{R} m q \quad , \quad \underline{z} = (\alpha_l \lambda + 1/\lambda)/\alpha_* = \left[(k+1)/2k\right] \sqrt{k_k / \varepsilon_v}\, z$$

$$f = \underline{q}\,\underline{z} = \left[(k+1)/2k\right] k_k (1/\varepsilon_v)^{\bar{k}_v / (\bar{k}_v - 1)} q z$$

$$Ma^2 = (\alpha_* \lambda / \sqrt{k_T \tau})^2 = (k_k / k_T \varepsilon_v) \lambda^2 / \tau \tag{8}$$

分别回归为第二小节的各式。

3. 以 k 组合数 ε 通连 β 系数气动函数式　文献 [4] 所用 β 系数，亦可分析如下：

$$\beta_h = k_T R / 2 \cdot (T^* - T)/(i^* - i) = \varepsilon_T$$

$$\beta_s = R l_n (T^* / T) / R l_n (\pi_T^{o*} / \pi_T^o) = (\bar{k} - 1)/\bar{k} (= \alpha_s) \tag{9}$$

于是各气动函数是：

$$\tau = 1/(1 + \beta_h Ma^2) = 1/(1 + \varepsilon_T Ma^2) \quad , \quad \pi = \tau^{1/\beta_s} = \tau^{\bar{k}/(\bar{k}-1)}$$

$$\underline{q} = \sqrt{k_T} Ma \tau^{(1/\beta_s) - (1/2)} = \lambda \tau^{1/(\bar{k}-1)} \sqrt{k_k / \varepsilon_v}$$

$$\underline{z} = \sqrt{\tau}\left[\sqrt{k_T} Ma + (1/\sqrt{k_T} Ma)\right] = \left[(k+1)/2k\right] \sqrt{k_k / \varepsilon_v}\, z \quad , \quad f = \underline{q}\,\underline{z} \tag{10}$$

也分别回归为第二小节各式。

结论　经过用 k 组合数 ε，简化了原解析解所用气动函数；连通了用 α、β 系数的气动函数式，说明它们都是以几个 k 值表示的变比热气动函数，因此也都可以用焓值拟合式解析求解，无需另作图解。$\underline{q}$ 函数省去了用 $\bar{k}_v$，$\underline{qz}$ 乘积求 f 函数也较简捷。以 Ma 数为自变数的气动函数式，竟只用到 k_T、k、$\bar{k}$。具体情况用 λ 或 Ma 好尚待工程实践总结。

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RESEARCH ON DISCHARGE COEFFICIENT OF COOLING HOLES ON FEEDER TUBE

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ABSTRACT The main air flow supplied in the feeder tube is delivered to cooling holes with axis perpendicular to the main flow. This crossflow pattern exerts a significant influence on the hole discharge coefficient. The experimental investigation of the hole discharge coefficients and the measurement of static pressure distribution along the tube axis have been completed. It is shown that the discharge coefficient increases as the ratio of the orifice thickness to diameter and the velocity head ratio between orifice flow and tube flow increase. But the discharge coefficient approaches to a constant when the velocity head ratio is greater than 100. Empirical formulas have been dedueed from the experimental data. The static pressure distribution along the tube and the air flow rate allocation for the holes have been calculated by using the general air system computer program developed by author. The results ageree with the experimental data well.

COORDINATION OF α、β COEFFICIENTS WITH k COMPLEXES FOR VARYING SPECIFIC HEAT SOLUTIONS

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ABSTRACT Three complexes ε_T, ε, ε_v of k are introduced to simplify the original expressions of gasdynamic functions with k of varying specific heats and their analytical solutions, and to unite two other forms of gasdynamic functions using coefficients α of β so that the calculations and applications at higher temperatures could be more convenient. It is clarified that the effect of varying specific heats reflected through k is the essence of these functions. The different sets of formulae are also briefly compared and analysed for reference in engineering applications.

NUMERICAL STUDY OF VISCOUS FLOWS IN COMPLEX GEOMETRIES

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ABSTRACT The program SIMPLEC has been adopted to calculate the viscous flows at all Mach numbers inside a plane cascad and nozzle. In order to tread both incompressible and compressible flows, pressure is chosen as a primary dependent variable in preference to density. The effect of the pressure on the density is implicitly included in the pressure correction equation. Further, the covariant velocity components are chosen as the dependent variables in the momentum equations, which makes the pressure-velocity coupling relatively easy to handle. Comparison of the calculated results with the experimental data show that the present method exibits much better stability and covergence behavior, and is applicable to a variety of the incompressible and compressible flows, ranging from subsonic to supersonic.